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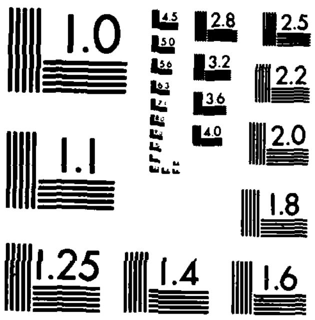

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19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The purpose of this interim scientific report is to summarize the Air Force sponsored research of Principal Investigators Moshe Goldberg and Marvin Marcus under Grant AFOSR-83-0150, during the period May 1, 1983 through April 30, 1984. The described efforts consist of the following projects: (a) Problems in stability analysis of finite difference approximations for hyperbolic initial-boundary value problems; (b) Matrix norms, condition numbers and the numerical solution of linear systems, and numerical range approximations. Such projects should contribute to better understanding of advanced computational techniques, and to the improvement of basic mathematical tools often used in numerical analysis and other fields of applied mathematics.

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INTERIM REPORT

Stability Analysis of Finite Difference Schemes for Hyperbolic Systems, and Problems in Applied and Computational Linear Algebra

AFOSR-83-0150

Principal Investigators: Marvin Marcus
Moshe Goldberg

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Table of Contents

ANNUAL INTERIM SCIENTIFIC TECHNICAL REPORT

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1. *Summary:*

The research is concerned with two principal related areas. Stability analysis of finite difference schemes for hyperbolic initial-boundary value problems lead to an investigation of bounds for matrix norms and condition numbers. The aim of this research is to provide a better understanding of tools used in the numerical analysis of such hyperbolic systems. Approximation schemes lead to systems of linear algebraic equations and the stability of such schemes depends on the eigenvalues and singular values of the associated matrices. Thus, in certain aspects, the analysis of a finite difference scheme is a problem in numerical linear algebra. Eigenvalue localization and inequalities for matrix norms are two pertinent areas of classical research in this field. The investigators have used methods from convex analysis, the theory of inequalities, classical linear and multi-linear algebra and numerical range theory in attacking these problems. This project should contribute to better understanding of advanced computational techniques, and to the improvement of basic mathematical tools often used in numerical analysis and other fields of applied mathematics.

2.3. *Research objectives and status of research*

The research completed by Moshe Goldberg under Air Force Grant AFOSR-83-0150 during May 1983 – April 1984, consists of the following two topics:

1. *Convenient Stability Criteria for Difference Schemes of Hyperbolic Initial-Boundary Value Problems*

Consider the first order system of hyperbolic partial differential equations

$$\partial u(x,t)/\partial t = A\partial u(x,t)/\partial x + Bu(x,t) + f(x,t), \quad x \geq 0, \quad t \geq 0,$$

where $u(x,t)$ is the unknown vector; A a hermitian matrix of the form $A = A_1 \oplus A_2$, where A_1 is negative definite and A_2 is positive definite; and $f(x,t)$ is a given vector. The problem is well posed in $L_2(0,\infty)$ if initial values

$$u(x,t) = u^0(x) \in L_2(0,\infty), \quad x \geq 0,$$

and boundary conditions

$$u_1(0,t) = Su_2(0,t) + g(t), \quad t \geq 0,$$

are prescribed. Here, u_1 and u_2 are the inflow and outflow parts of u corresponding to the partition of A, and S is a coupling matrix.

In the past year, E. Tadmor and M. Goldberg, [16], have succeeded in obtaining new, easily checkable stability criteria for a wide class of finite difference approximations for the above initial-boundary value problem. The difference approximations consist of a general difference scheme -- explicit or implicit, dissipative or not, two-level or multi-level – and boundary conditions of a rather general type.

Attention is restricted to the case where the outflow boundary conditions are translatory, i.e., determined at all boundary points by the same coefficients.

This, however, is not a severe limitation since such boundary conditions are commonly used in practice. In particular, when the numerical boundary consists of a single point, the boundary conditions are translatory by definition.

Throughout the paper [16] it is assumed that the basic scheme is stable for the pure Cauchy problem, and that the assumptions which guarantee the validity of the stability theory of Gustafsson, Kreiss and Sundstrom [18] hold. With this in mind the question of stability for the entire difference approximation is raised.

The first step in the stability analysis was to prove that the approximation is stable if and only if the scalar outflow components of its principal parts are stable. This reduces the global stability question to that of a scalar homogeneous outflow problem of the form

$$\partial u / \partial t = a \partial u / \partial x, \quad a = \text{constant} > 0, \quad x \geq 0, \quad t \geq 0$$

$$u(x,0) = u^0(x), \quad x \geq 0; \quad u(0,t) = 0, \quad t \geq 0.$$

The stability criteria obtained in [16] for the reduced problem depend both on the basic difference scheme and on the boundary conditions, but very little on the interaction between the two. Such criteria eliminate the need to analyze the intricate and often complicated interaction between the basic scheme and the boundary conditions, hence providing in many cases convenient alternatives to the well known stability criteria of Kreiss [22], and of Gustafsson, Kreiss and Sundstrom [18]. It should be pointed out that the old scheme-independent stability criteria in [13,14] easily follow from the present criteria in [16].

Having the new criteria in [16], all the examples in the previous papers [13,14] were reestablished. For instance, if the basic scheme is arbitrary (dissipative or not) and the boundary conditions are generated by either the explicit or implicit right-sided Euler schemes, then overall stability is assured. For dissipative basic schemes stability is proved if the boundary conditions are determined by either oblique extrapolation, the Box-scheme, or by the right-sided weighted Euler scheme. These and other examples incorporate most of the cases discussed in recent literature [2], [3], [13], [14], [18], [19], [21], [23], [27], [30-35], [37].

Some new examples appear in [16] as well. Among these it is found that if the basic scheme is arbitrary and two-level, then horizontal extrapolation at the boundary maintains overall stability. Other stable cases occur when the basic scheme is given by either the backward (implicit) Euler scheme or by the Crank-Nicolson scheme, and the boundary conditions are determined by oblique extrapolation. Such examples, where neither the basic scheme nor the boundary conditions are necessarily dissipative, could not have been handled by the previous results in [13,14].

An extended version of [16], which includes additional examples and remarks, is now in final stages of preparation, [17].

Such contributions should be helpful to applied mathematicians and engineers in better understanding and exploiting old and new finite difference approximations to hyperbolic systems.

2. *Submultiplicativity and Other Properties of l_p Norms for Matrices*

In the past three years, E. G. Straus (now deceased) and M. Goldberg, [9,10], investigated submultiplicativity properties of norms and seminorms on operator algebras -- an important subject in many fields of numerical analysis and applied mathematics. In this work an arbitrary normed vector space V over the complex field C, with an algebra $L(V)$ are studied. If N is positive definite, i.e.,

$N(A) > 0$ for all $A \neq 0$, then N is called a generalized operator norm. If in addition, N is (sub-) multiplicative, namely $N(AB) \leq N(A)N(B)$ for all $A, B \in L(V)$, then N is called an operator norm on $L(V)$.

Given a seminorm N on $L(V)$ and a fixed constant $\mu > 0$, then obviously $N_\mu \equiv \mu N$ is a seminorm too. Similarly, N_μ is a generalized operator norm if and only if N is. In both cases, N_μ may or may not be multiplicative. If it is, then μ is said to be a multiplicativity factor for N.

Having these definitions the following is proved in [9]:

(i) If N is a nontrivial seminorm or a generalized operator norm on $L(V)$, then N has multiplicativity factors if and only if

$$\mu_N \equiv sup\{N(AB) : N(A) = N(B) = 1\} < \infty.$$

(ii) If $\mu_N < \infty$, then μ is a multiplicativity factor for N if and only if $\mu \geq \mu_N$.

Special attention was given to the finite dimensional case where it suffices, of course, to consider $C_{n \times n}$, the algebra of $n \times n$ complex matrices. Following Ostrowski, [28], in this case the terms generalized matrix norm and matrix norm are adopted instead of generalized operator norm and operator norm, respectively. In this case it is proved that while nontrivial, indefinite seminorms on $C_{n \times n}$ never have multiplicativity factors, generalized matrix norms always have such factors. In the infinite dimensional case, however, the situation was less decisive, i.e., there exist nontrivial indefinite seminorms and generalized operator norms on $L(V)$ which may or may not have multiplicativity factors.

In both the finite and infinite-dimensional cases it is proved that if M and N are seminorms on $L(V)$ such that M is multiplicative, and if $\eta \geq \zeta > 0$ are constants satisfying

$$\zeta M(A) < N(A) < \eta M(A) \quad \text{for all } A \in L(V),$$

then any μ with $\mu > \eta / \xi^2$ is a multiplicativity factor for N.

Using these results it is proved, for example, that if V is an arbitrary Hilbert space and

$$r(A) = sup\{|(Ax,x)| : x \in V, \ |x| = 1\}, \quad A \in L(V),$$

is the classical numerical radius, then μr is an operator norm if and only if $\mu \geq 4$. This assertion is of interest since the numerical radius r is perhaps the best known nonmultiplicative generalized operator norm [1,4,15,20,29], and it plays an important role in stability analysis of finite difference schemes for multi-space-dimensional hyperbolic initial-value problems [15,24,25,36].

Straus and Goldberg also investigated C-numerical radii which constitute a generalization of the classical numerical radius r, defined in [7] as follows: For given matrices $A, C \in C_{n\times n}$, the C-numerical radius of A is

$$r_C(A) = \max\{|tr(CU^*AU)| : U \ \ n\times n \text{ unitary}\}.$$

In [7] (compare [26]), it is shown that r is a norm on $C_{n\times n}$ -- and so has multiplicativity factors -- if and only if C is not a scalar matrix and $tr\ C \neq 0$. Multiplicativity factors for the above r_C were found in [7-10,12].

In the most recent effort, Straus and Goldberg, [11], studied the well known l_p norms

$$|A|_p = \{\textstyle\sum_{ij} |a_{ij}|^p\}^{1/p}, \quad A = (a_{ij}) \in C_{n\times n}, \ 1 \leq p \leq \infty.$$

It was shown by Ostrowski, [28], that these norms are multiplicative if and only if $1 \leq p \leq 2$. For $p \geq 2$ it is shown that μ is a multiplicativity factor for $|A|_p$ if and only if $\mu \geq n^{1-2/p}$; thus, in particular, obtaining the useful result that $n^{1-2/p}|A|_p$ is a multiplicative norm on $C_{n\times n}$.

Continuing this effort, Goldberg obtained [5,6] the best possible constants $\mu(p,q)$ and $\mu(q,p)$ (for arbitrary $1 \le p,q \le \infty$) such that relations of the form

$$|AB|_p \le \mu(p,q)|A|_p|B|_q, \quad |AB|_p \le \mu(q,p)|A|_q|B|_p$$

hold whenever the matrix product AB exists. This leads to the best possible constant $\lambda(p,q)$ for which

$$\|A\|_p \le \lambda(p,q)|A|_q, \quad A \in C_{m\times n}, \; (1 \le p,q \le \infty),$$

where

$$\|A\|_p = \max\{|Ax|_p : x \in C^n, \; |x|_p = 1\}$$

is the ordinary l_p operator norm of an arbitrary $m \times n$ matrix A. Such inequalities could be useful in determining the power boundedness of a matrix -- a basic question in stability analysis.

The research of Marvin Marcus under Air Force Grant AFOSR-83-0150 during May 1983 -- April 1984 is described below.

In the mathematical modelling of physical phenomena, a standard technique for solving the resulting partial differential equation boundary value problem is to approximate the differential system at a discrete set of points by the solution of a linear system. In general, such a system is large, non-symmetric and sparse. The Tchebychev iteration based on Tchebychev polynomials can be used to solve non-symmetric linear systems whose eigenvalues lie in the right half-plane. Moreover, many factorizations and splitting techniques applied to symmetric systems yield non-symmetric systems with spectra in the right-half plane. Manteuffel [Numer. Math. 28, 307-327, 1977] showed that the Tchebychev iteration for an N-square real linear system

$$Ax = b$$

whose eigenvalues lie in the right-half plane, can be carried out with two parameters, c, d which arise in evaluating $T_n(d/c)$ where $T_n(z) = \cosh(n\cosh^{-1}(z))$ is the n^{th} Tchebychev polynomial.

Manteuffel proved that if the convex hull, $H(A)$, of the spectrum of A is known (for normal matrices this is the numerical range of A), then the parameters c, d in the above iteration can be chosen to be optimal in a minimax sense. In a later paper [Numer. Math. 31, 183-208, 1978] he discusses a method of estimating $H(A)$ during iteration from the sequence of residual vectors. He also shows that a power method variant of the Tchebychev procedure yields eigenvalue estimates that lie in the numerical range $F(A)$. B. A. Carré, L. A. Hagemann, R. B. Kellog, and others, have also done work on dynamic estimation of the optimal SOR parameter.

At the 30th Anniversary Meeting of the Society for Industrial & Applied Mathematics in 1982, (supported in part by a grant from the Air Force Office of Scientific Research), Dr. Martin Schultz of the Yale University Department of Computer Science posed the following general question stemming from the work of Manteuffel and its continuation by the Yale group.

Obtain computational estimates of the set $H(A)$ and the field of values (i.e., numerical range) $F(A)$.

In work currently underway by M. Marcus and M. Sandy, computer codes have been developed for obtaining explicit numerical plots of $H(A) \subset F(A)$ that are useful in determining the optimal c, d described above. Work is also in progress to obtain computable bounds on the discrepancy between $F(A)$ and $H(A)$. This discrepancy problem has been considered earlier in various forms by many authors: H. Wielandt, B. N. Moyls, I. Filippenko, B. Shure, M. Sandy, K. Fan, C. A. Berger, M. Newman, R. C. Thompson, P. R. Halmos.

Another of the areas of research under this grant is concerned with the condition number $\kappa(A)$ of a matrix A, defined by

$$\kappa(A) = \|A\| \, \|A^{-1}\|,$$

which plays an important role in error estimates and termination criteria for numerical iterative methods for the solution of linear systems. More specifically, the following problem was examined in publication #2 listed under *Publications* below.

How does the condition number $\kappa(A \cdot B)$ of the Hadamard product

$$A \cdot B = [a_{ij} \, b_{ij}]$$

of two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$, depend on the norms, singular values, etc. of the factors A and B?

In particular, work appearing in the preceding publication investigates this and related problems for the class of von Neumann norms. Recall that a von Neumann norm, $\|X\|$, also called a unitarily invariant norm, satisfies

$$\|UXV\| = \|X\|$$

for all unitary matrices U and V. von Neumann's theorem states that if $\alpha_1 \geq \cdots \geq \alpha_n$ are the singular values of X, then $\|X\|$ can be written as

$$\|X\| = \varphi(\alpha_1, \ldots, \alpha_n)$$

where φ is a symmetric gauge function. Note that the usual l_p − norms arise from gauge functions:

$$\varphi(x_1, \ldots, x_n) = \max_{\sigma \in S_n} \left[\sum_{i=1}^{k} |x_{\sigma(i)}|^p \right]^{1/p}$$

where k is a fixed integer, $1 \leq k \leq n$, and $p \geq 1$, e.g., if $k=1$ and $p=1$ then φ specializes to the Hilbert norm or maximum singular value of X

$$\alpha_1 = \max_{\|x\|=1} \|Xx\|.$$

In the preceding formula $\|x\|$ is the usual Euclidean norm. In other words,

$$\varphi(x_1, \ldots, x_n) = \max_{\sigma \in S_n} |x_{\sigma(1)}|$$

which obviously satisfies the definition of a symmetric gauge function.

It is worthwhile to note that there exists a simple connection between the condition number $\kappa(A)$ and the Hadamard product. It is proved in publication #2 listed below, that for the Hilbert norm the following inequality is available:

$$\|A \cdot B\| \leq \|A\| \cdot \|B\|.$$

In case $B = A^{-1}$ then

$$\|A \cdot A^{-1}\| \leq \|A\| \cdot \|A^{-1}\|.$$

Thus for the usual matrix norm subordinate to the Euclidean vector norm

$$\kappa(A) \geq \|A \cdot A^{-1}\|.$$

It follows easily that

$$\kappa(A) \geq L$$

where L is the largest product

$$L = \max_{i,j} |a_{ij}\ b_{ij}|,$$

and $B = A^{-1}$. Hence, in particular, if a lower bound for the modulus of a single element of A^{-1} is available, say $|b_{i_0 j_0}| \geq L_0$, then

$$\kappa(A) \geq |a_{i_0 j_0}| L_0.$$

These preliminary results suggest a number of possible areas of investigation which are currently underway:

Investigate the inequality

$$\|A \cdot B\| \leq \|A\| \cdot \|B\|$$

for the class of von Neumann norms and obtain easily computable lower bounds on the corresponding condition numbers.

For the linear system $Ax = b$ these considerations show that the estimated relative error in x can be as large as

$$\frac{\|A \cdot A^{-1}\|}{1 - \|A \cdot A^{-1}\|} (\alpha + \beta),$$

where α and β are the relative errors in A and b respectively.

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Marvin Marcus

1. Marcus, M., Kidman, K. and Sandy, M., Products of Elementary Doubly Stochastic Matrices, in press, Linear and Multilinear Algebra, V. 15, pp. 331-340, 1983.

2. Marcus, M., Kidman, K. and Sandy, M., Unitarily Invariant Generalized Matrix Norms and Hadamard Products, in press, Linear and Multilinear Algebra, 1983.

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8. Marcus, M. and Sandy, M., Interior points of generalized numerical ranges, in preparation.

Moshe Goldberg

1. On the mapping $A \to A^+$, Linear and Multilinear Algebra 12 (1983), 285-289.

2. Combinatorial inequalities, matrix norms, and generalized numerical radii. II (with E. G. Straus), in "General Inequalities 3", edited by E. F. Beckenbach and W. Walter, Birkhauser Verlag, Basel, 1983, 195-204.

3. Multiplicativity of l_p norms for matrices (with E. G. Straus), Linear Algebra and Its Applications 52 (1983), 351-360.

4. Multiplicativity factors for C-numerical radii (with E. G. Straus), Linear Algebra and Its Applications 54 (1983), 1-16.

5. On generalizations of the Perron-Frobenius Theorem (with E. G. Straus), Linear and Multilinear Algebra 14 (1983), 143-156.

6. New stability criteria for difference approximations of hyperbolic initial-boundary value problems (with E. Tadmor), in "Lectures in Applied Mathematics Vol. 22," American Mathematical Society, accepted.

7. Multiplicativity of l_p norms for matrices. II, Linear Algebra and Its Applications, accepted.

8. Some inequalities for l_p norms of matrices, in "General Inequalities 4", edited by W. Walter, Birkhauser-Verlag, Basel, accepted.

9. In Memoriam Edwin F. Beckenbach, in "General Inequalities 4", edited by W. Walter, Birkhauser-Verlag, Basel, accepted.

10. Convenient stability criteria for difference approximations of hyperbolic

initial-boundary value problems (with E. Tadmor), Mathematics of Computation, accepted.

5. *List of Professional Personnel, Advanced Degrees*

Marvin Marcus, Professor of Mathematics and Computer Science, University of California, Santa Barbara.

Moshe Goldberg, Professor of Mathematics, Technion, Israel Institute of Technology.

Kent Kidman, awarded Ph.D, Fall 1983, currently working for Hughes Aerospace, Tactical software division, El Segundo, CA, thesis title: Stochastic Matrices and Unitarily Invariant Norms.

Markus Sandy, Research Assistant, graduate student in Department of Mathematics.

Interactions.

- M. Sandy represented the investigators on this grant at AFOSR Conference on supercomputing, Air Force Weapons Laboratory, April 3-6, 1984.
- M. Marcus invited to speak at Mathematics Department Seminar, University of California, San Diego, May 22, 1984.
- M. Marcus invited March 3, 1984 to American Math. Soc. Joint Summer Research Conference at Bowdoin College, on Linear Algebra and Systems Theory.
- M. Goldberg invited speaker, (two talks), the Fourth International Conference on General Inequalities, Mathematical Research institute, Oberwolfach, West Germany, May 1983.
- M. Goldberg invited speaker, the Fifteenth AMS-SIAM (American Mathematical Society and Society for Industrial and Applied Mathematics) Summer Seminar on Large-Scale Computations in Fluid Mechanics, Scripps Institute

of Oceanography, University of California, San Diego, La Jolla, California, June-July 1983.

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